Berry's Phase Induced Bose-Einstein Condensation into a Vortex State

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The existence of a geometric phase in magnetic traps can be used to Bose condense a magnetically trapped atomic gas into a vortex state. We propose an experimental setup where a magnetic trap together with a blue detuned laser beam form a multiply connected trap geometry. The local variation of the magnetic quantization axis induces a geometric or Berry's phase that allows the atoms to acquire an effective gauge charge interacting with the analog of a magnetic solenoid. It is shown that the ground state of such a system may be given by a vortex state. We also discuss the influence of atomic interactions on the proposed vortex production scheme in the context of present Bose-Einstein condensation experiments with dilute gases.

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The first experimental realization of Bose-Einstein condensation in dilute alkali gases [1–3] has dramatically revived the interest in weakly interacting Bose gases. Compared to previous work, the spatial inhomogeneity and finite size of the new experimental systems as well as the possibility to study complicated nonequilibrium dynamics have added many interesting conceptual elements and have even shed new light on some basic paradigms of Bose-Einstein condensation. By now, also these novel Bose condensed systems seemed to be fairly well understood. However, as we will show in this paper, further surprises may still be abound.

From the very beginning, the main focus of the investigation of Bose condensed alkali gases concentrated on the condensate itself, the macroscopically occupied ground state of the system. According to a credo of many textbooks on quantum mechanics (cf. [4]) the ground state of a particle in a spherically or axially symmetric potential has vanishing angular momentum. This statement is usually derived from the famous theorem of nodelessness of the ground state [5] that in turn is based on the assumption that the eigenstates of the Hamiltonian can always be chosen to be real. However, this assumption does not necessarily hold for a charged particle in the presence of a magnetic field [6]. In our paper we show, that the action of an effective gauge field on neutral atoms in magnetic traps, caused by a geometric phase effect, can lead to a situation where the *ground* state of the system has a nonvanishing angular momentum and the gas can directly Bose condense into a vortex state. In so far, our vortex production scheme differs drastically from previous proposals that suggested the creation of a macroscopically occupied *excited* state of the system by opto-mechanical stirring [7], adiabatic population transfer [8] or the accidental generation of vortices in a quench [9,10].

Before discussing the specific details of our proposed trap configuration, we want to illustrate in terms of classical physics the basic mechanism that is used to create a ground state with nonvanishing angular momentum. Consider an electrically charged pendulum in the absence of gravity whose pivot point is pierced by an infinite solenoid perpendicular to the pendulum's plane of oscillation. The Hamiltonian for the pendulum then reads

$$\mathcal{H} = \frac{MR^2\dot{\phi}^2}{2} = \frac{(\mathcal{L}_z - \Delta\mathcal{L}_z)^2}{2I} \ , \tag{1}$$

where M and R are the mass and length of the pendulum respectively. $I = MR^2$ is the momentum of inertia of the pendulum, while $\mathcal{L}_z = MR^2\dot{\phi} + \Delta\mathcal{L}_z$ is the angular momentum, canonically conjugated to the phase ϕ. The shift $\Delta\mathcal{L}_z$ of the angular momentum is given by

$$\Delta\mathcal{L}_z = \frac{q\,\Phi_B}{2\pi c} \ , \tag{2}$$

where q is the pendulum charge, c is the speed of light, and Φ_B is the magnetic flux across the solenoid.

Apparently, the ground state of the Hamiltonian (1) posesses a nonvanishing canonical angular momentum $\mathcal{L}_z = \Delta\mathcal{L}_z$. In classical mechanics this increase of the canonical angular momentum does not lead to any immediately observable physical effect, since the angular frequency $\dot{\phi} = 0$ of the pendulum is the same as in the absence of a magnetic field. Only when the magnetic field within the solenoid is switched off, the conserved canonical angular momentum is transformed into mechanical angular momentum and the pendulum is set into motion. Microscopically this generation of mechanical angular momentum is a consequence of the displacement current that is induced by a time-dependent magnetic field due to the Maxwell equation $\boldsymbol{\nabla} \times \mathbf{E} = -(1/c)\dot{\Phi}_B$. In quantum mechanics, however, the canonical angular momentum is directly observable without altering the magnetic field. Here, the shift $\Delta\mathcal{L}_z$ can be made visible by an interference experiment similar to the one used to display the Aharonov-Bohm effect [11].

With this brief illustration we have shown how the existence of a magnetic field can lead to a ground state with nonvanishing angular momentum. The direct use of the outlined idea in present experiments with Bose gases is

however flawed by the electric neutrality of atoms. Instead, we show that the local variation of the hyperfine spin orientation in the inhomogeneous field of a magnetic trap leads to a geometric phase effect that can be interpreted as the presence of an effective magnetic field.

Consider an atom in a hyperfine state F, which moves in an external magnetic field $B(\mathbf{r})$. The Hamiltonian of the system is given by

$$\mathcal{H} = \frac{\hat{\mathbf{p}}^2}{2M} + \hat{V}(\mathbf{r}) \tag{3}$$

$$\hat{V}(\mathbf{r}) = g\,\mu_B\,\hat{\mathbf{F}}\cdot\mathbf{B}(\mathbf{r}), \tag{4}$$

with the atomic mass M, the Bohr magneton μ_B, and the Lande factor g.

If the atomic motion is slow enough, the evolution of the system can be described using the Born-Oppenheimer approximation. The latter assumes that for every point $\mathbf{r}$ the internal state of the atoms is close to a local eigenstate $|\tilde{m}(\mathbf{r})\rangle$

$$\hat{V}(\mathbf{r})|\tilde{m}(\mathbf{r})\rangle = \tilde{m}|\tilde{m}(\mathbf{r})\rangle \tag{5}$$

of the magnetic contribution $\hat{V}(\mathbf{r})$ to the Hamiltonian. The total wave function can then be written

$$\langle\mathbf{r},m|\Psi\rangle = \Psi_{\tilde{m}}(\mathbf{r})\langle m|\tilde{m}(\mathbf{r})\rangle \tag{6}$$

where $|m\rangle$ corresponds to an eigenstate of the projection operator $F_z = \mathbf{e}_z\cdot\hat{\mathbf{F}}$ of the hyperfine spin on the z-axis.

Applying the full Hamiltonian Eq. (3) to the wave function $\langle\mathbf{r},m|\Psi\rangle$ of Eq. (6) shows that the evolution of the adiabatic wave function $\Psi_{\tilde{m}}$ is governed by the Born-Oppenheimer Hamiltonian [12]

$$H = \frac{[\hat{\mathbf{p}} - \mathbf{A}_{\tilde{m}}(\mathbf{r})]^2}{2M} + V_{\tilde{m}}(\mathbf{r}) + \phi_{\tilde{m}}(\mathbf{r})\,, \tag{7}$$

with the magnetic potential

$$V_{\tilde{m}}(\mathbf{r}) = g\,\mu_B\,\tilde{m}\,|\mathbf{B}(\mathbf{r})|, \tag{8}$$

the vector gauge potential

$$\mathbf{A}_{\tilde{m}}(\mathbf{r}) = i\hbar\langle\tilde{m}|\nabla\tilde{m}\rangle, \tag{9}$$

and the scalar gauge potential

$$\phi_{\tilde{m}}(\mathbf{r}) = \frac{\hbar^2\sum_{\tilde{m}'\neq\tilde{m}}|\langle\tilde{m}'|\nabla\tilde{m}\rangle|^2}{2M}. \tag{10}$$

In addition to the phase change caused by the kinetic energy and the potential energy $V_{\tilde{m}}(\mathbf{r})$ the adiabatic wave function $\Psi_{\tilde{m}}(\mathbf{r})$ acquires the Berry's phase [13]

$$\gamma_{\tilde{m}} = \frac{1}{\hbar}\oint_\Gamma d\mathbf{r}\cdot\mathbf{A}_{\tilde{m}}(\mathbf{r}) \tag{11}$$

on a semiclassical trajectory along the closed path Γ.

However, according to Eq. (6) there exists an ambiguity in the definition of the adiabatic wave function $\Psi_{\tilde{m}}(\mathbf{r})$ and of the gauge potentials. The eigenstate equation (5) defines the local eigenstate $|\tilde{m}(\mathbf{r})\rangle$ only up to an arbitrary gauge transformation $|\tilde{m}(\mathbf{r})\rangle \longrightarrow \exp[i\varphi_{\tilde{m}}(\mathbf{r})]\,|\tilde{m}(\mathbf{r})\rangle$. To eliminate this ambiguity we specify the gauge by choosing the following definition for $|\tilde{m}(\mathbf{r})\rangle$:

$$|\tilde{m}(\mathbf{r})\rangle = \hat{\mathcal{R}}\left(\frac{[\mathbf{e}_z\times\mathbf{B}(\mathbf{r})]}{|[\mathbf{e}_z\times\mathbf{B}(\mathbf{r})]|}, \mathrm{ang}(\mathbf{e}_z,\mathbf{B}(\mathbf{r}))\right)|m=\tilde{m}\rangle. \tag{12}$$

The operator $\hat{\mathcal{R}}(\mathbf{n},\Theta)$ describes a three-dimensional rotation around an axis $\mathbf{n}$ by the angle Θ. The state $|m=\tilde{m}\rangle$ is an eigenstate of the hyperfine spin projection on the z-axis with eigenvalue $\tilde{m}$.

Our next task is to evaluate the potentials of Eqs. (8), (9), and (10) for some particular magnetic trap. We choose a two-dimensional version of the Ioffe-Pritchard trap

$$\mathbf{B}(\mathbf{r}) = \lambda x\,\mathbf{e}_x - \lambda y\,\mathbf{e}_y + B_0\,\mathbf{e}_z, \tag{13}$$

assuming that the atomic motion is constrained to the xy-plane by some auxiliary external field that is independent of the internal hyperfine spin state. We will discuss a particular version of such a confinement below. We also restrict ourselves to the internal hyperfine state $F=1$, $\tilde{m}=-1$, $g<0$ that corresponds to present experiments with trapped sodium gases. The discussion of the general case is postponed to the end of the paper [14]. Using polar coordinates for the xy-components of $\mathbf{r}$, we obtain the expressions

$$V_{-1}(\rho) = -g\,\mu_B\sqrt{B_0^2+\lambda^2\rho^2}, \tag{14}$$

$$\mathbf{A}_{-1}(\boldsymbol{\rho}) = \frac{\hbar\,\Delta\mathcal{L}_{-1}(\rho)}{\rho}\,\mathbf{e}_\varphi(\boldsymbol{\rho}), \tag{15}$$

$$\Delta\mathcal{L}_{-1}(\rho) = -1 + \frac{B_0}{\sqrt{B_0^2+\lambda^2\rho^2}}, \tag{16}$$

and

$$\phi_{-1}(\rho) = \frac{2B_0^2\,\lambda^2 + \lambda^4\rho^2}{4(B_0^2+\lambda^2\rho^2)^2}, \tag{17}$$

with $\boldsymbol{\rho} = x\,\mathbf{e}_x + y\,\mathbf{e}_y$ and $\mathbf{e}_\varphi(\boldsymbol{\rho}) = (-y\,\mathbf{e}_x + x\,\mathbf{e}_y)/\rho$. In the special case of a vanishing bias field $B_0=0$, the angular momentum shift is equal to $\Delta\mathcal{L}_{-1}=-1$ and the gauge vector field $\mathbf{A}_{-1}(\boldsymbol{\rho})$ is equivalent to the electromagnetic vector field of a solenoid along the z-axis with $q\,\Phi_B/c = -2\pi$. In so far, the situation of the trapped atoms resembles the previous example of the electrically charged pendulum interacting with a solenoid. We therefore conjecture that the ground state of the trapped atom gas has a nonvanishing canonical angular momentum. However, before coming to a definitive conclusion about the ground state of the system, we have to add a few

elements to the model outlined in Eq. (13) in order to make it realistic.

The system Hamiltonian (7) does not yet contain a longitudinal confinement along the z-direction. We therefore suggest to add a red-detuned light sheet parallel to the xy-plane. It has to be created by a far detuned and linearly polarized light beam in order to achieve an ac Stark shift that is independent of the direction of the hyperfine spin $\mathbf{F}$. To a good approximation such a dipole trap can be described by a harmonic potential with oscillation frequency ω_z.

Since it is our plan to create a vortex in the absence of a bias field B_0, we have to find a way to suppress atom losses due to Majorana spin flips in the center of the trap. Therefore another, far blue-detuned light beam should be added that essentially plugs the region with the most critical trap losses (cf. [2]). This linearly polarized light beam is assumed to have a potential energy $V_{\rm plug}(\rho) = V_{\rm plug,0}\exp(-2\rho^2/w_{\rm plug}^2)$ with beam waist $w_{\rm plug}$.

Finally, we have to account for interactions between the gas atoms. For dilute gases this can be done by adding the mean-field pressure

$$V_{\rm mf}(\mathbf{r}) = gN|\Psi_{-1}(\mathbf{r})|^2 \tag{18}$$

to the Hamiltonian (7). The coupling constant $g = 4\pi\hbar^2 a/M$ is given by the s-wave scattering length a, while N is the total number of atoms. Furthermore we assume the axial confinement to be much stronger than the atomic interaction energy ($\hbar\omega_z \gg gN|\Psi_{-1}|^2$). In this case the atomic wave function

$$\Psi_{-1}(\mathbf{r}) = \psi_{-1}(\boldsymbol{\rho})\cdot\chi^{(0)}(z) \tag{19}$$

factorizes into a radial part ψ_{-1} and the ground state of the axial potential $\chi^{(0)}(z)$. The radial wave function $\psi_{-1}(\boldsymbol{\rho})$ is then determined by a two-dimensional Gross-Pitaevskii Hamiltonian with an effective coupling strength $g_{\rm 2D} = g\sqrt{M\omega_z}/\sqrt{2\pi\hbar}$. Note, however, that the existence of such an effectively two-dimensional situation is not instrumental for the proposed vortex creation. It was solely assumed to facilitate a numerical treatment of the problem.

As a consequence, the ground state of the system is given as a solution of the two-dimensional Gross-Pitaevskii equation

$$H_{\rm GP}\,\psi_{-1} = \mu\,\psi_{-1}, \tag{20}$$

with the nonlinear Hamilton operator

$$\begin{aligned} H_{\rm GP} = {} & \frac{[\hat{\mathbf{p}} - \mathbf{A}_{-1}(\boldsymbol{\rho})]^2}{2M} + V_{-1}(\rho) + \phi_{-1}(\rho) \\ & + V_{\rm plug}(\rho) + g_{\rm 2D}N|\psi_{-1}(\boldsymbol{\rho})|^2 + \frac{\hbar\omega_z}{2}, \end{aligned} \tag{21}$$

the chemical potential μ, the total atom number N. Our goal is to prove numerically that the minimum of the system energy

$$E[\psi,\psi^\star] = \langle\psi|NH_{\rm GP} - \frac{g_{\rm 2D}\,N^2|\psi_{-1}|^2}{2}|\psi\rangle. \tag{22}$$

is reached for a state with nontrivial value of the angular momentum $\mathcal{L}_z = -1$. Using the separability of the radial and angular motion, we divide the energy minimization procedure into two steps: first, we minimize the energy under the constraint of a particular value of the angular momentum $\mathcal{L}_z$, then we compare the obtained energies in order to find its global minimum.

As it was expected, we found that for the case of a vanishing bias field $B_0 = 0$, the energy minimum is reached for the $\mathcal{L} = -1$ state. As an illustration, we plot the energy difference between the $\mathcal{L} = -1$ and $\mathcal{L} = 0$ state in Fig. 1. One can see that for a broad range of condensate populations the energy of the vortex state is constantly lower than the energy of the axially symmetric state. As a further illustration, Fig. 2a shows the radial distribution of the atomic density of the vortex state. We regard this as a convincing proof of a possible Bose-Einstein condensation into a vortex state. It is no surprise, however, that the described effect has not yet been observed experimentally, since for typically used large bias fields of $B_0 \gg \lambda\bar{\rho}$, $\bar{\rho}$ being a typical cloud radus, the condensation into a vortex state is energetically not favored.

The main inconvenience of the proposed setup lies in the potential difficulties that it poses for the detection of a vortex state. The density distributions for different angular momenta are almost indistinct, since the most obvious signature of vorticity – a hole in the center of the density distribution – is masked by the optical plug. We therefore suggest the following experimental sequence in order to make the vorticity directly observable.

- Turn on slowly a bias field B_0 that is strong enough to prevent trap losses by Majorana spin flips. This stage is equivalent to turning off the solenoid field in the case of the pendulum [15].
- Remove the optical plug adiabatically, by lowering the light intensity (cf. Fig. 2b). Then, remove the magnetic confinement instantaneously and allow the cloud to expand freely within the xy-plane.

A detectable hole in the middle of the density distribution can now be taken as a distinctive feature of a vortex state. Alternatively, the vorticity of the state can be determined by an interference experiment [16].

In conlusion, we have pointed out that magnetically trapped atoms are subject to an effective gauge potential that is related to the existence of a geometric phase. Under certain circumstaces, the trapped atoms behave as if they were charged particles interacting with a magetic field of a flux carrying solenoid. We show that this effective magnetic field can yield a ground state of the system with nonvanishing angular momentum, and thereby open the possibility of a direct Bose-Einstein condensation into a vortex state. Our results can be easily generalized to

an arbitrary value of the hyperfine spin F and to different kinds of magnetic traps. Indeed, one can show that the angular momentum of the ground state is then given by

$$\mathcal{L}_z = -\tilde{m}\Upsilon, \tag{23}$$

where Υ is the topological index of the used magnetic field configuration. It measures in terms of 2π the rotation angle of the magnetic field vector $\mathbf{B}(\mathbf{r})$ along a closed path around the center of the trap. The proposed trap geometry of this paper corresponds to a topological index of $\Upsilon = -1$. In contrast, a quadrupole trap combined with an additional red-detuned light sheet in the xy-plane would correspond to an index $\Upsilon = +1$.

After this work has been materially completed, we learned about a related vortex production scheme employing the Ahoronov-Casher effect [17]. It combines a trap geometry very similar to ours with an electrically charged wire. In principle, such a scheme would allow the controlled creation of arbitrary vortex states, even though the experimental requirements seem to be challenging. We show, in contrast, that Bose-Condensation into a vortex state may already occur in a more basic trap setup without the charged wire.

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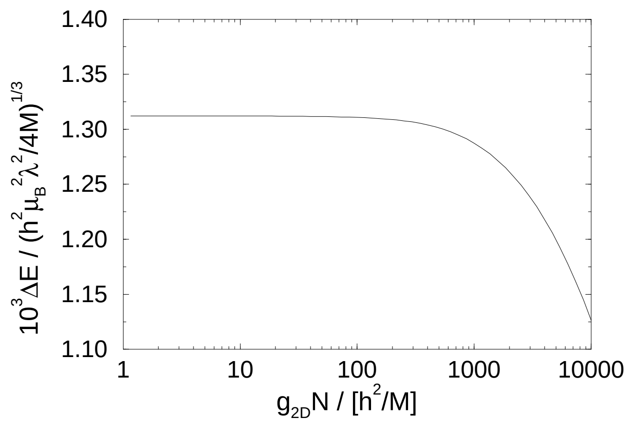

FIG. 1. In this figure, we plot for sodium atoms the energy difference $\Delta E = E_{\mathcal{L}_z=0} - E_{\mathcal{L}_z=-1}$ between energy minimizing states with constrained angular momenta $\mathcal{L}_z = 0$ and $\mathcal{L}_z = -1$ as a function of the atomic interaction strength $g_{2\mathrm{D}}N$. We use the two-dimensional Ioffe-Pritchard trap with optical plug and confining light sheet that is described in the body of this paper. We use the realistic set of trap parameters $\lambda = 223\,\mathrm{Gs/cm}$, $w = 3.05\,\mu\mathrm{m}\,(= 10 \times (2\hbar^2/\mu_B\lambda M)^{1/3})$, $V_{\mathrm{plug},0} = 2\pi \times \hbar \times 9.7\mathrm{MHz}$, and $\omega_z = 26\,\mathrm{kHz}$. For this set of parameters, the interaction strength $g_{2D}NM/\hbar^2 = 1000$ corresponds to $N = 1250$ atoms and an axial Thomas-Fermi parameter of $g_{2\mathrm{D}}\bar{n}_{2\mathrm{D}}/\hbar\omega_z \approx 0.5$, where $\bar{n}_{2\mathrm{D}}$ is the peak value of the two-dimensional density in the xy-plane.

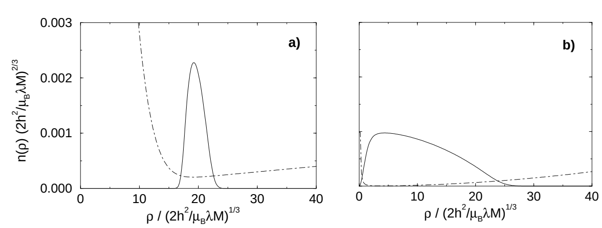

FIG. 2. Here, we show the normalized two-dimensional density distribution $n(\rho)$ for a vortex state with $\mathcal{L}_z = -1$ and 1250 sodium atoms ($g_{2D}NM/\hbar^2 = 1000$). The left plot corresponds to the interacting ground state of the toroidal trap described in the caption of Fig. 1. The dot-dashed line shows the effective potential energy including magnetic field, light plug, gauge potentials, and the cenrifugal barrier. After the Bose-Einstein condensation into the vortex state is completed, a bias field $B_0 = 2\,\mathrm{Gs}$ is slowly turned on and the optical plug is slowly removed. The resulting trap configuration consists of a two-dimensional Ioffe-Pritchard trap with radial frequency $\omega_\perp = 391\,\mathrm{Hz}$, which corresponds to 0.08 in the units of Fig. 1, and the axial light sheet described in the caption of Fig. 1. The right figure demonstrates that the resulting atomic density distribution vanishes for small ρ as a consequence of the nonvanishing centrifugal barrier of a vortex state.